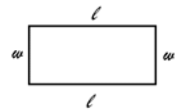
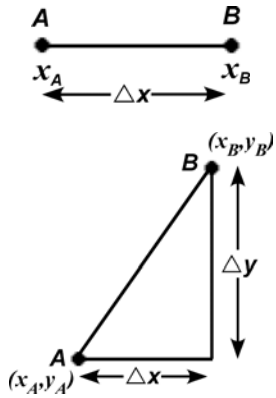
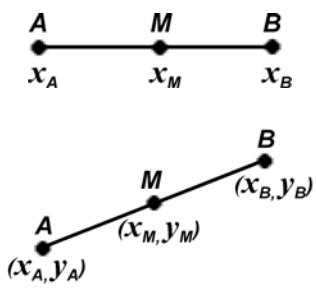
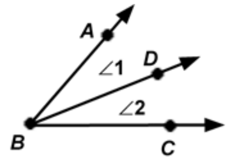
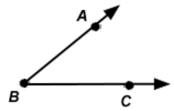
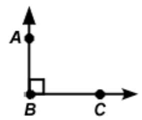
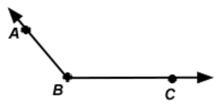
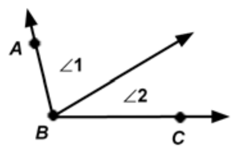
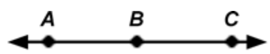
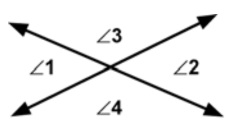
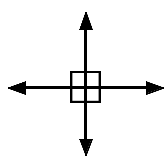
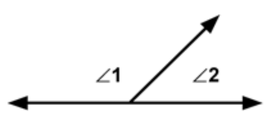
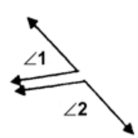
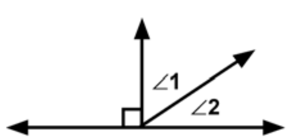
## Geometry Patterns, Chapter 1 - Chapter 9

***Patterns from Chapter 1***

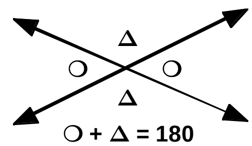
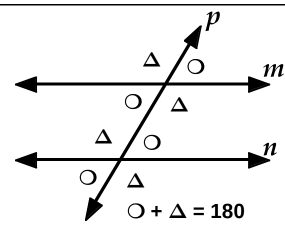
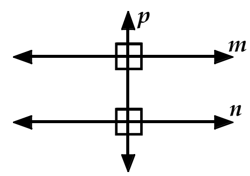
| Pattern Name | Picture | Formalism |
|---|---|---|
| 1-3:<br>**Rectangle** |  | $\boxed{Perimeter = 2l + 2w}$<br>$\boxed{Area = l \cdot w}$ |
| 1-4:<br>**The "measure function" *m*()**<br>***m*(*some geometry object*) *is not a multiplication*. It has a *real number* value which is the "measure" of the geometry object.** | **Uses of *m*():**<br>$m(\overline{AB})$ =length of segment $\overline{AB}$<br>(units are length units)<br><br>*m*(angle)=degrees in angle<br>*m*(arc)=degrees in arc | **"Segment Measure" or "Segment Length"**<br>$m(\overline{AB})$ is written $AB$.<br>It is the distance between the end points $A$ and $B$. |
| 1-4:<br>**Segment Addition Postulate** |  | IF $Q$ is between $P$ and $R$<br>THEN $PQ + QR = PR$ |
| 1-4:<br>**Distance Formula**<br><br>**for calculating segment length.**<br>(The formula for the real line and for a plane might look different but they are actually the same; one term from the plane formula is just zero in the real line case.) |  | "$\Delta x$" means:<br>"the difference in the x-coordinates"<br>$\Delta x = (x_B - x_A)$<br>$\Delta y = (y_B - y_A)$<br>On a line: $\boxed{AB = \lvert\Delta x\rvert}$<br>On a coordinate plane:<br>$\boxed{AB = \sqrt{\Delta x^2 + \Delta y^2}}$ |
| 1-5:<br>**Segment midpoint**<br><br>**The midpoint of a segment is a point on the segment equally distant from the segment's endpoints.**<br>(On a line the coordinate of the midpoint is the average of the coordinates of the endpoints. In the coordinate plane just separately figure the *x*-coordinate of the midpoint from the *x*-coordinates of the endpoints; do the same thing for the *y*-coordinate of the midpoint.) |  | Definition of midpoint:<br>*AM* = *MB*.<br>On a line: $x_M = \frac{x_A + x_B}{2}$<br>On a coordinate plane:<br>$\boxed{x_M = \frac{x_A + x_B}{2}}$<br>and<br>$\boxed{y_M = \frac{y_A + y_B}{2}}$ |

| | | |
|---|---|---|
| 1-6:<br>**Angle Bisector**<br>**Ray $\overrightarrow{BD}$ bisects $\angle ABC$ means:**<br>**$\angle ABD$ and $\angle DBC$ have equal measures. This means by definition: $\angle ABD \cong \angle DBC$** |  | For an angle bisector:<br>$\boxed{m\angle 1 = m\angle 2}$ |
| 1-6:<br>**Classifying Angles by their Measures:** | <br><br> | **Acute angle: measure < 90º**<br><br>**Right angle: measure = 90º**<br><br>**Obtuse angle:**<br>**90º < measure < 180º** |
| 1-6:<br>**Angle Addition Postulate**<br>***Adjacent angles* share a common ray; one angle is not in the interior of the other.** |  | For adjacent angles:<br>$\boxed{m\angle 1 + m\angle 2 = m\angle ABC}$ |
| 1-6:<br>**Straight Angle**<br>**Two opposing rays.** |  | For a straight angle:<br>$\boxed{m\angle ABC = 180°}$ |
| 1-7:<br>**Vertical Angles are congruent**<br><br>**A pair of intersecting lines forms two opposite pairs of vertical angles. Vertical angles have equal measures and are therefore congruent.**<br>Corollary:<br>**Perpendicular lines intersect to form four right angles.** | <br> | When two lines cross:<br>Two vertical angles:<br>$\boxed{m\angle 1 = m\angle 2}$<br>$\boxed{m\angle 3 = m\angle 4}$<br>Four linear pairs:<br>$\boxed{m\angle 1 + m\angle 3 = 180°}$<br>$\boxed{m\angle 3 + m\angle 2 = 180°}$<br>$\boxed{m\angle 2 + m\angle 4 = 180°}$<br>$\boxed{m\angle 4 + m\angle 1 = 180°}$ |
| 1-7:<br>**Linear Pair**<br>**Two adjacent angles are a linear pair when they form a straight angle.**<br>$\angle 1$ and $\angle 2$ are also supplementary |  | For a linear pair:<br>$\boxed{m\angle 1 + m\angle 2 = 180°}$ |
| 1-7:<br>**Supplementary Angles**<br>**Two angles are supplementary if their measures sum to 180º.**<br>$\angle 1$ and $\angle 2$ need not be adjacent. |  | For supplementary angles:<br>$\boxed{m\angle 1 + m\angle 2 = 180°}$ |
| 1-7:<br>**Complementary Angles**<br>**Two angles are complementary if their measures sum to 90º.**<br>$\angle 1$ and $\angle 2$ need not be adjacent. |  | For complementary angles:<br>$\boxed{m\angle 1 + m\angle 2 = 90°}$ |

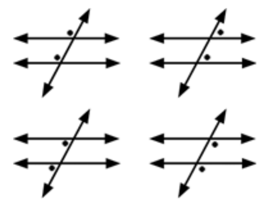
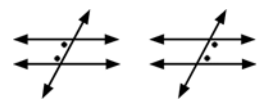
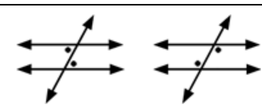
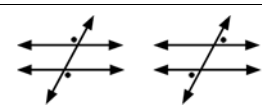
| Propositions and IF…THEN… Statements | | |
|---|---|---|
| A **Proposition** is any statement that is either **true** or **false** | | |
| A **Conditional Statement** (or **IF…THEN…** statement) is itself a proposition. | Form of the Conditional Statement: (***Hypothesis*** and ***conclusion*** can be any propositions)<br>**IF** *hypothesis* **THEN** *conclusion* | Conditional Statements are the tools you use to make new true propositions. You use them this way:<br>**IF** the hypothesis is true<br>**THEN** the conclusion is true |
| **Constructing New Propositions** | | |
| We describe the two **Construction Rules** this way | (▲, ■, and ● are placeholders for any propositions)<br>**Inputs:**<br>If you have these two<br>- - - - - - - - - - - - - - - - - - -<br>**Result:**<br>You may then write this | |
| The **Detachment** Construction Rule | **IF ▲ THEN ■**<br>**▲**<br>— - - - - - - - - - - —<br>**■** | Conditions for using Detachment:<br>1. An IF…THEN…statement<br>2. Any proposition<br>3. The proposition matches the hypothesis of IF…THEN…statement<br>- - - - - - - - - - - - - - - - - - - - - - - - -<br>Then you can state by itself the (detached) ■ conclusion of the IF…THEN…statement.<br>Notice that the matching ▲s do not appear in the result. |
| The **Syllogism** Construction Rule | **IF ▲ THEN ■**<br>**IF ■ THEN ●**<br>— - - - - - - - - - - —<br>**IF ▲ THEN ●** | Conditions for using Syllogism:<br>1. A first IF…THEN…statement<br>2. A second IF…THEN…statement<br>3. Conclusion of first IF…THEN…statement matches hypothesis of second IF…THEN…statement<br>- - - - - - - - - - - - - - - - - - - - - - - - -<br>Then you can state a new IF ▲ THEN ● statement using the hypothesis of the first and the conclusion of the second.<br>Notice that the matching ■s do not appear in the result. |

| Properties of Equality of Real Numbers<br>In the following, $a$, $b$, $c$, and $d$ can stand for any real number, except that $d \neq 0$. | | |
|---|---|---|
| **Reflexive Property** | $a = a$ | Every number equals itself. |
| **Symmetric Property** | **IF** $a = b$ **THEN** $b = a$ | You can switch the two sides of an equation. |
| **Transitive Property** | **IF** $a = b$ **and** $b = c$<br>**THEN** $a = c$ | You can chain equations together leaving out the middle parts, if the middle parts are the same. |
| **Subtraction and Addition Properties** | **IF** $a + b = c$ **THEN** $a = c - b$<br><br>**IF** $a - b = c$ **THEN** $a = c + b$ | You can subtract the same number from both sides.<br>You can add the same number to both sides.<br>***You can jump a term over the equal sign if you change its sign***. (Terms are numbers that are being added or subtracted.) |
| **Division and Multiplication Properties** | **IF** $d \bullet a = b$ **THEN** $a = \frac{b}{d}$<br><br>**IF** $\frac{a}{d} = b$ **THEN** $a = b \bullet d$ | You can divide both sides by the same *nonzero* number.<br>You can multiply both sides by the same number.<br>***You can jump a nonzero factor over the equal sign if you invert it (change it to its reciprocal)***.<br>(Factors are numbers that are being multiplied or divided.) |
| **Substitution Property** | **IF** $a = b$ **THEN** $a$ and $b$ are interchangeable in any expression or equation | Numbers with equal values can be substituted for each other anywhere. (Notice that this implies the transitive property.) |
| **Properties of Multiplication and Addition** | | |
| **Distributive Property** | $a \bullet (b + c) = a \bullet b + a \bullet c$ | Factors get distributed across terms inside the parentheses. |
| **Commutative Properties** | $a + b = b + a$<br>$a \bullet b = b \bullet a$ | You can reverse two added terms or two multiplied factors. |
| **Associative Properties** | $a + (b + c) = (a + b) + c$<br>$a \bullet (b \bullet c) = (a \bullet b) \bullet c$ | In a chain of additions or a chain of multiplications, it doesn't matter which operations you do first. |

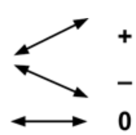
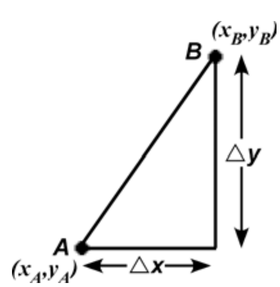
***Chapter 3: Transversals and angles***

| | | |
|---|---|---|
| **"Coconuts"**<br>The intersection of two lines creates a cluster of four angles. |  | **○ + Δ = 180** |
| **"Double Coconuts"**<br>A transversal intersecting two lines creates two clusters of four angles each.<br>In these formulas it is assumed that $m \parallel n$ . If you know any one angle you know all eight. |  | **○ + Δ = 180**<br>All acute angles are congruent (equal).<br>All obtuse angles are congruent (equal).<br>Any obtuse and any acute angle are supplementary (they total 180). |
| **"Perpendicular Double Coconuts"**<br>$m \parallel n$ and $p \perp n$ (or $p \perp m$ ) |  | If any one angle is 90° then all eight angles are 90°. |

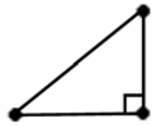
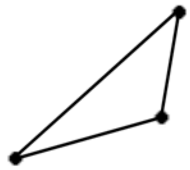
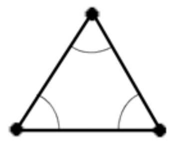
***3-1, 3-2: Angle pairs***

| | | |
|---|---|---|
| **Corresponding Angles** |  | Corresponding angles are congruent (equal). |
| **Consecutive Interior Angles** |  | Consecutive Interior angles are supplementary (they total 180). |
| **Alternate Interior Angles** |  | Alternate Interior angles are congruent (equal). |
| **Alternate Exterior Angles** |  | Alternate Exterior angles are congruent (equal). |

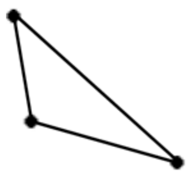
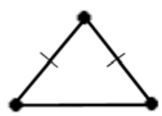
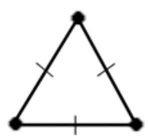
***3-3: Slope***

| | | |
|---|---|---|
| **Sign of Slope**<br>Figure the absolute value $\left\|\frac{\Delta y}{\Delta x}\right\|$ and then attach the sign. |  | Positive rises to the right.<br>Negative falls to the right.<br>Horizontal line has zero slope.<br>Vertical line has undefined slope. |
| **The Slope Formula**<br>Or use the slope formula to get both sign and value at once. |  | "Δ" is not a number; it is pronounced "delta" and means "the change in":<br>$\Delta y = y_B - y_A$<br>$\Delta x = x_B - x_A$<br>The ***slope formula***:<br>Slope of $\overline{\mathrm{AB}} = \frac{\Delta y}{\Delta x}$ |

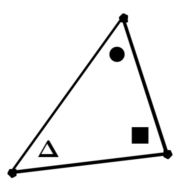
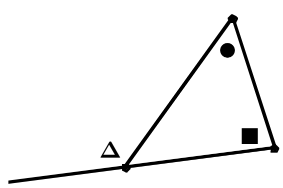
***4-1: Classify Triangles by their Angles***

| | | |
|---|---|---|
| **Acute Triangle** |  | All three angles are acute (they have measures less than 90°). |
| **Right Triangle** |  | One angle is a right (90°) angle. There can not be more than one right angle in a triangle. |
| **Obtuse Triangle** |  | One angle is obtuse (it has a measure greater than 90°). There can not be more than one obtuse angle in a triangle. |
| **Equiangular Triangle** |  | All three angles are congruent (equal). |

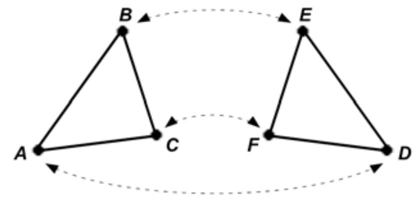
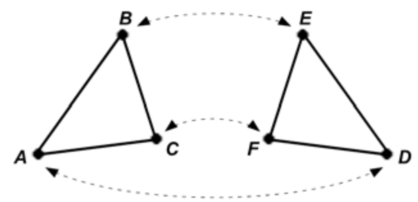
***4-1: Classify Triangles by their Sides***

| | | |
|---|---|---|
| **Scalene Triangle** |  | No two sides are congruent. (All sides are different.) |
| **Isosceles Triangle** |  | At least two sides are congruent (equal). |
| **Equilateral Triangle** |  | All three sides are congruent (equal). (An equilateral triangle is also an isosceles triangle.) |

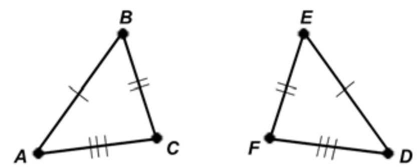
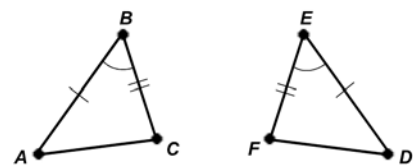
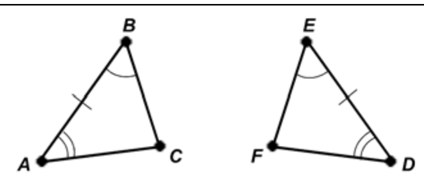
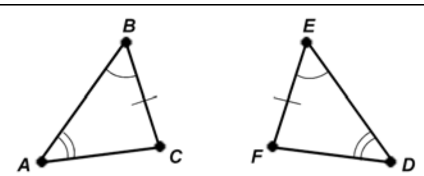
***4-2: Measuring Angles in Triangles***

| | | |
|---|---|---|
| **Angle Sum Theorem** |  | **Δ + ● + ■ = 180°**<br>The sum of the measures of the interior angles of a triangle is 180º. |
| **Exterior Angle Theorem** |  | **Δ = ● + ■**<br>The measure of an *exterior* angle of a triangle is equal to the sum of the measures of the two *remote* interior angles. |

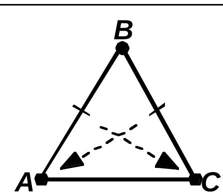
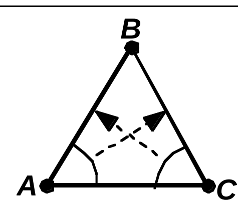
### *4-3: Congruent Triangles*

| | | |
|---|---|---|
| **Correspondence Between Two Triangles**<br>($\leftrightarrow$ means "corresponds to") |  | $\Delta ABC \leftrightarrow \Delta DEF$<br>means *all* of these:<br>$\angle A \leftrightarrow \angle D$   $\overline{AB} \leftrightarrow \overline{DE}$<br>$\angle B \leftrightarrow \angle E$   $\overline{BC} \leftrightarrow \overline{EF}$<br>$\angle C \leftrightarrow \angle F$   $\overline{CA} \leftrightarrow \overline{FD}$ |
| **Congruence Statement**<br>**C.P.C.T.C.: Corresponding Parts of Congruent Triangles are Congruent** (definition of triangle congruence) |  | $\mathbf{\Delta ABC \cong \Delta DEF}$ means:<br>$\Delta ABC \leftrightarrow \Delta DEF$<br>and *all* of these:<br>$\angle A \cong \angle D$   $\overline{AB} \cong \overline{DE}$<br>$\angle B \cong \angle E$   $\overline{BC} \cong \overline{EF}$<br>$\angle C \cong \angle F$   $\overline{CA} \cong \overline{FD}$ |

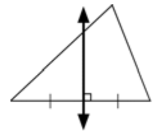
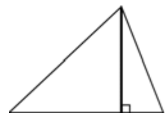
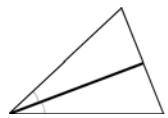
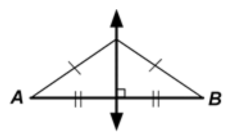
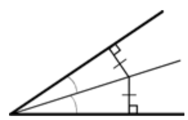
### *4-4, 4-5: Congruence Postulates and Theorem*

| | | |
|---|---|---|
| **SSS Postulate**<br>Side-Side-Side |  | If the **three sides** of one triangle are congruent to the three sides of a second triangle, then the two triangles are congruent. |
| **SAS Postulate**<br>Side-Angle-Side |  | If **two sides and the included angle** of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent. |
| **ASA Postulate**<br>Angle-Side-angle |  | If **two angles and the included side** of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent. |
| **AAS Theorem**<br>Angle-Angle-Side |  | If **two angles and a *non-included* side** of one triangle are congruent to the ***corresponding*** two angles and side of a second triangle, then the two triangles are congruent. |

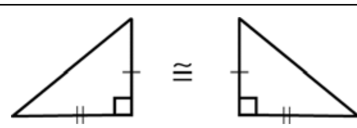
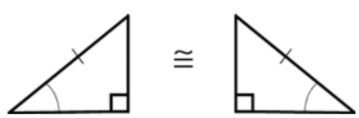
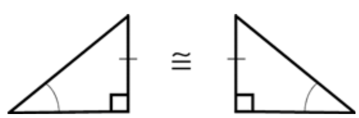
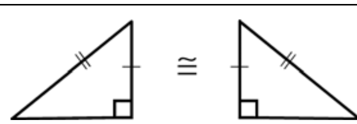
### *4-6: Isosceles Triangle Theorems*

| | | |
|---|---|---|
| (At least)<br>**Two Sides Congruent**<br>then opposite angles congruent |  | Stated in terms of number equality:<br>**IF** $AB = BC$ **THEN** $m\angle C = m\angle A$<br>Stated in terms of congruence:<br>**IF** $\overline{AB} \cong \overline{BC}$ **THEN** $\angle C \cong \angle A$ |
| (At least)<br>**Two Angles Congruent**<br>then opposite sides congruent |  | Stated in terms of number equality:<br>**IF** $m\angle C = m\angle A$ **THEN** $AB = BC$<br>Stated in terms of congruence:<br>**IF** $\angle C \cong \angle A$ **THEN** $\overline{AB} \cong \overline{BC}$ |

## *5-1 Special Segments in Triangles*

| | | |
|---|---|---|
| Definition: **Perpendicular Bisector of a Triangle** |  | A Perpendicular Bisector of a Triangle is a line or segment perpendicular to a side of the triangle that intersects the side at its midpoint. |
| Definition: **Median of a Triangle** |  | A Median of a Triangle is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side. |
| Definition: **Altitude of a Triangle** |  | An Altitude of a Triangle is a segment perpendicular to a side of the triangle, one of whose endpoints is the vertex of the angle opposite that side and the other of whose endpoints is on the line containing the side. |
| Definition: **Angle Bisector of a Triangle** |  | An Angle Bisector of a Triangle is a segment that bisects an angle of the triangle, one of whose endpoints is the vertex of the angle and the other of whose endpoints is on the side opposite that angle. |
| **Theorems 5-1, 5-2:** (5-1 and 5-2 are converses of each other) |   | 5-1: Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment. 5-2: Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment. |
| **Theorems 5-3, 5-4:** (5-3 and 5-4 are converses of each other) |  | 5-3: Any point on the bisector of an angle is equidistant from the sides of the angle. 5-4: Any point on or in the interior of an angle and equidistant from the sides of an angle lies on the bisector of the angle. |

## *5-2 Right Triangles*

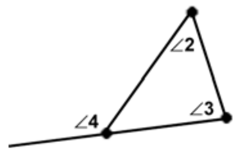
| | | |
|---|---|---|
| **Theorem 5-5: LL Congruence** |  | If the two legs of one right triangle are congruent, respectively, to the two legs of another right triangle, then the triangles are congruent. |
| **Theorem 5-6: HA Congruence** |  | If the hypotenuse and an acute angle of one right triangle are congruent, respectively, to the hypotenuse and an acute angle of another right triangle, then the triangles are congruent. |
| **Theorem 5-7: LA Congruence** |  | If one leg and an acute angle of one right triangle are congruent, respectively, to a leg and ***corresponding*** acute angle of another right triangle, then the triangles are congruent. |
| **Postulate 5-1: HL Congruence** |  | If the hypotenuse and a leg of one right triangle are congruent, respectively, to the hypotenuse and a leg of another right triangle, then the triangles are congruent. |

## Properties of Inequality of Real Numbers

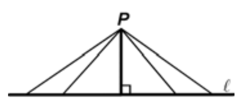
In the following, $a$, $b$, and $c$ can stand for any real number, except as noted.

| | | |
|---|---|---|
| **Definition of Inequality** | $a < b$ | $a < b$ means that $a$ is ***to the left of*** $b$ on the real line.<br>$a < b$ if and only if there is a *positive* number $c$ such that $a + c = b$.<br>$a < b$ and $b > a$ are the same statement. |
| | $a > b$ | $a > b$ means that $a$ is ***to the right of*** $b$ on the real line.<br>$a > b$ if and only if there is a *positive* number $c$ such that $a = b + c$ .<br>$a > b$ and $b < a$ are the same statement. |
| **Comparison Property** | $a < b$, $a = b$, or $a > b$ | Exactly one of these three must be true. |
| **Transitive Property** | **IF** $a < b$ **and** $b < c$ **THEN** $a < c$<br><br>**IF** $a > b$ **and** $b > c$ **THEN** $a > c$ | You can chain the same inequalities together leaving out the middle parts, if the middle parts are the same number. |
| **Addition and Subtraction Properties** | **IF** $a > b$ **THEN** $a + c > b + c$ and $a - c > b - c$<br><br>**IF** $a < b$ **THEN** $a + c < b + c$ and $a - c < b - c$ | You can subtract the same number from both sides.<br>You can add the same number to both sides.<br>***You can jump a term over the equal sign provided that you change its sign.*** (Terms are numbers that are being added or subtracted.) |
| **Multiplication and Division Properties** | **IF** $c > 0$ and $a < b$ **THEN** $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$<br><br>**IF** $c > 0$ and $a > b$ **THEN** $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$<br><br>**IF** $c < 0$ and $a < b$ **THEN** $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$<br><br>**IF** $c < 0$ and $a > b$ **THEN** $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$ | You can multiply and divide both sides by the same *positive* number.<br>You can multiply and divide both sides by the same *negative* number ***provided that you reverse the inequality***. |

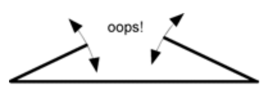
*5-3 Indirect Proof*

| | | |
|---|---|---|
| **Steps for Writing an Indirect Proof** | **IF** $h$ **THEN** $c$ | 1. Assume that the conclusion $c$ is false.<br>2. Show that this assumption leads to a contradiction of the hypothesis $h$ or of some other proposition known to be true.<br>3. The assumption must be incorrect; therefore the conclusion $c$ is true. |
| **Theorem 5-8: Exterior Angle Inequality Theorem** |   | If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its remote interior angles.<br>$m\angle 4 > m\angle 2$ ; $m\angle 4 > m\angle 3$ |

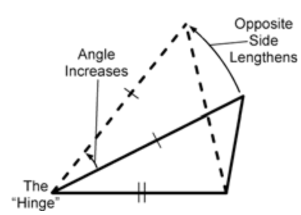
*5-4 Inequalities for Sides and Angles of a Triangle*

| | | |
|---|---|---|
| **Theorems 5-9, 5-10:**<br>(5-9 and 5-10 are converses of each other.)<br>The Bigger the Side, the Bigger the Opposite Angle.<br>The Bigger the Angle, the Bigger the Opposite Side |   | 5-9: If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.<br>5-10: If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle. |
| **Theorem 5-11:**<br>A Perpendicular Line is the Shortest Route from a Point to a Line |   | The perpendicular segment from a point to a line is the shortest segment from the point to the line. |

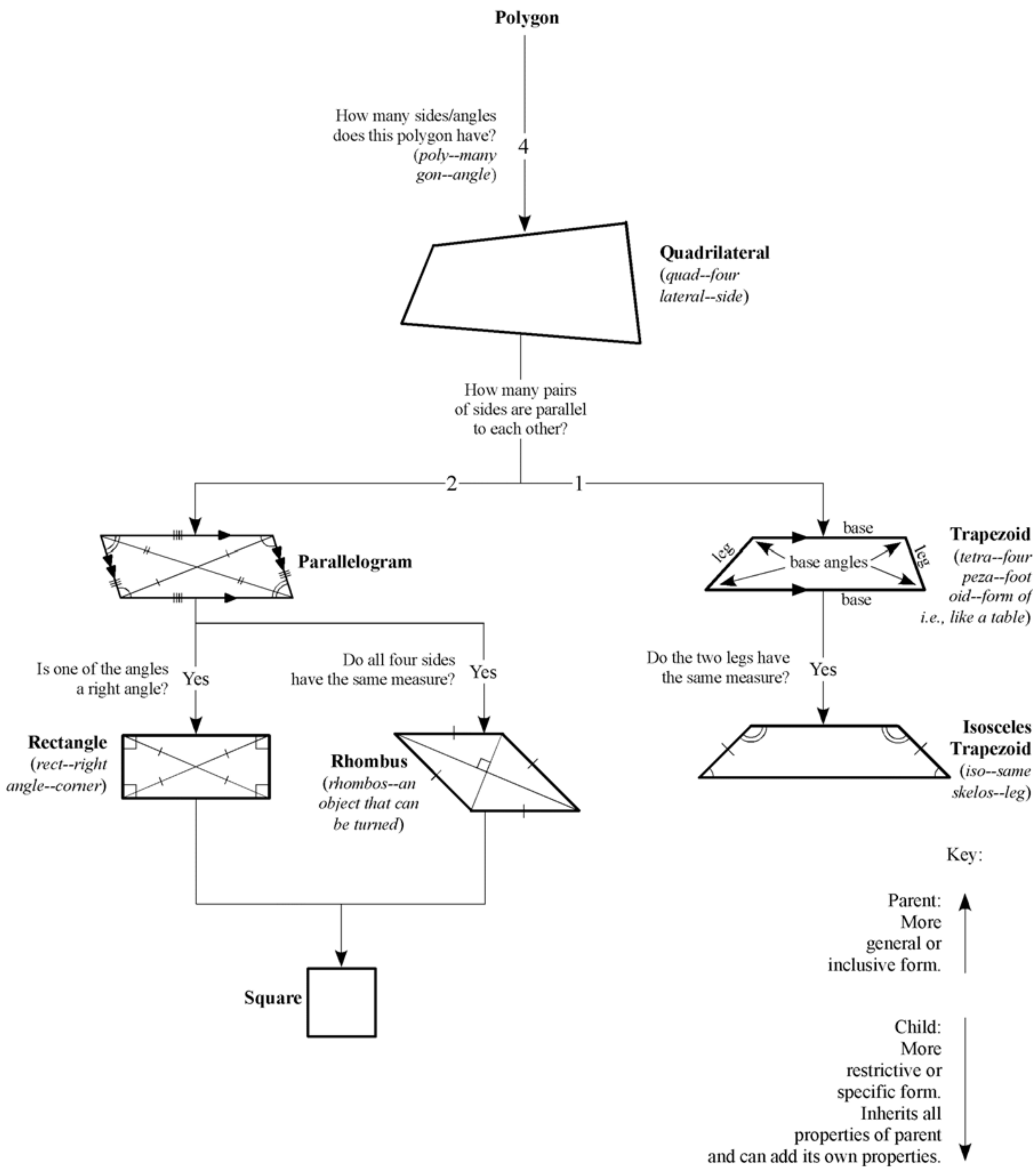
*5-5 The Triangle Inequality*

| | | |
|---|---|---|
| **Theorem 5-12: Triangle Inequality Theorem** |   | Given any two sides of a triangle, the sum of the lengths of these two sides is greater than the length of the third side.<br>Given a triangle any two of whose sides measure ***a*** and ***b***, the third side, ***c***, must be between these limits:<br>$\lvert a-b\rvert < c < (a+b)$<br>In particular, if ***a*** and ***b*** are the longest two of the three sides,<br>$\lvert a-b\rvert < c$ |

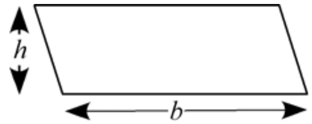
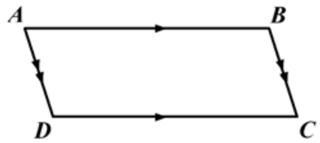
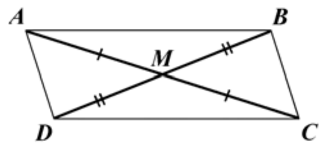
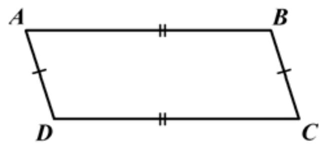
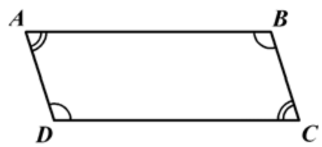
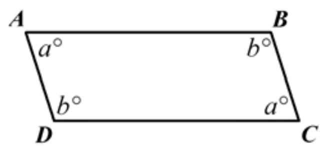
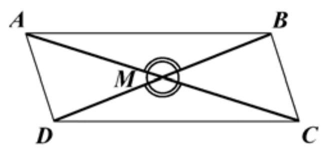
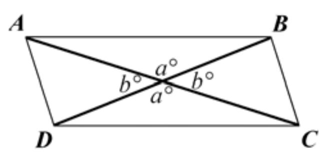
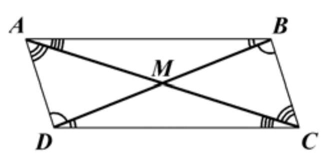
*5-6 Inequalities Involving Two Triangles*
*(if two sides of one are respectively congruent to two sides of the other)*

| | | |
|---|---|---|
| **Theorems 5-13, 5-14:**<br>**5-13: “SAS Inequality (Hinge) Theorem”**<br>**5-14: “SSS Inequality Theorem”**<br>(Despite their different names, 5-13 and 5-14 are converses of each other.) |   | 5-13: If two sides of one triangle are congruent, respectively, to two sides of another triangle, and the included angle in the first triangle is greater than the included angle in the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.<br>5-14: If two sides of one triangle are congruent, respectively, to two sides of another triangle, and the third side of the first triangle is longer than the third side of the second triangle, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle. |

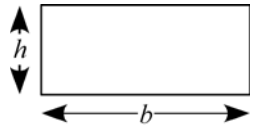
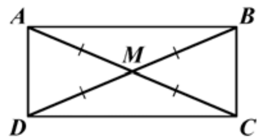
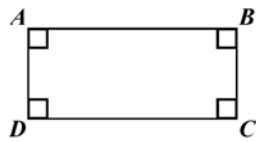
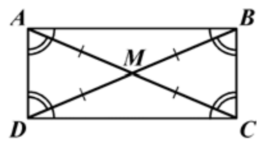
## Quadrilateral Family Tree



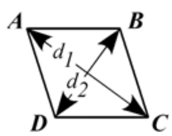
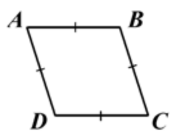
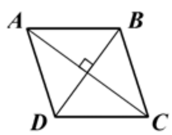
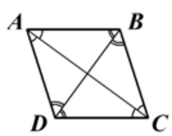
***Parallelogram***

| | | |
|---|---|---|
| **1. Area** |   | $A = b \bullet h$ |
| **2. Opposite Sides are Parallel** |   | $\overline{AB} \parallel \overline{CD}$, $\overline{DA} \parallel \overline{BC}$ |
| **3. Each Diagonal Bisects the Other** |   | $AM = MC$<br>$DM = MB$<br>M is the midpoint of $\overline{AC}$ and of $\overline{BD}$ |
| **4. Opposite Sides are Congruent** |   | $AB = CD$<br>$DA = BC$ |
| **5. Opposite Vertex Angles are Congruent** |   | $m\angle A = m\angle C$<br>$m\angle B = m\angle D$ |
| **6. Consecutive Vertex Angles are Supplementary** |   | $a + b = 180$<br>$b = 180 - a$<br>$a = 180 - b$ |
| **7. Vertical Angles Formed by the Intersection of Diagonals are Congruent** |   | $m\angle AMD = m\angle CMB$<br>$m\angle BMA = m\angle DMC$ |
| **8. Adjacent Angles Formed by the Intersection of Diagonals are Supplementary** |   | $a + b = 180$<br>$b = 180 - a$<br>$a = 180 - b$ |
| **9. Alternate Interior Angles Formed by the Diagonals as Trans-versals Intersecting the Sides are Congruent** |   | $m\angle MDA = m\angle MBC$<br>$m\angle MDC = m\angle MBA$<br>$m\angle MAD = m\angle MCB$<br>$m\angle MAB = m\angle MCD$ |

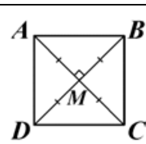
***Rectangle (Rectangle inherits properties from Parallelogram)***

| | | |
|---|---|---|
| **10. Area** |   | $A = b \bullet h$ |
| **11. The Diagonals are Congruent, and the Semi-diagonals are Mutually Congruent** |   | $AC = BD$<br>$AM = BM = CM = DM$<br>M is the midpoint of $\overline{AC}$ and of $\overline{BD}$ |
| **12. Vertex Angles are Right Angles** |   | $m\angle A = m\angle B = m\angle C = m\angle D = 90°$ |
| **13. Triangles Formed By Diagonals and Sides Are Isosceles** |   | $MA = MB$, $m\angle MAB = m\angle MBA$<br>(etc. for 3 more triangles) |

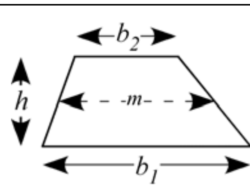
***Rhombus (Rhombus inherits properties from Parallelogram)***

| | | |
|---|---|---|
| **14. Area** |   | $A = \frac{1}{2} d_1 \bullet d_2$ |
| **15. The Four Sides are Mutually Congruent** |   | $AB = BC = CD = DA$ |
| **16. The Diagonals are Perpendicular to Each Other** |   | $\overline{AC} \perp \overline{BD}$ |
| **17. The Diagonals Bisect the Vertex Angles** |   | $m\angle CAB = m\angle CAD = m\angle ACB = m\angle ACD$<br>$m\angle DBA = m\angle DBC = m\angle BDA = m\angle BDC$ |

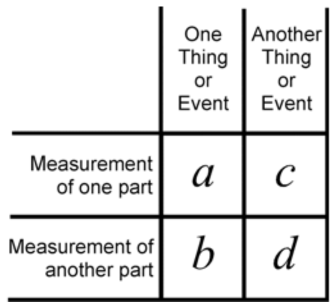
***Square (Square inherits properties from both Rectangle and Rhombus)***

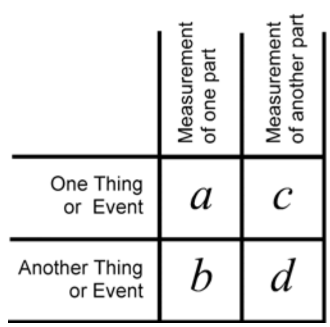
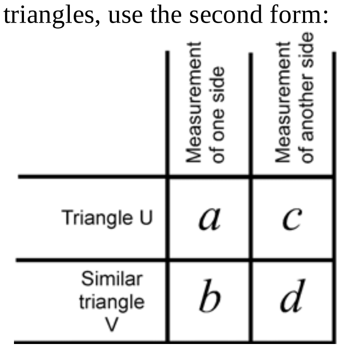
| | | |
|---|---|---|
| **18. Diagonals and Sides Form 45°- 45°- 90° Right Triangles** |   | $AB = \sqrt{2} \bullet AM$ (etc.)<br>$AC = \sqrt{2} \bullet AB$ (etc.) |

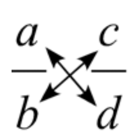
***Trapezoid***

| | | |
|---|---|---|
| **19. Area** |   | $A = \frac{1}{2} h(b_1 + b_2)$<br>(or: $m = \frac{1}{2}(b_1 + b_2)$; $A = m \bullet h$) |

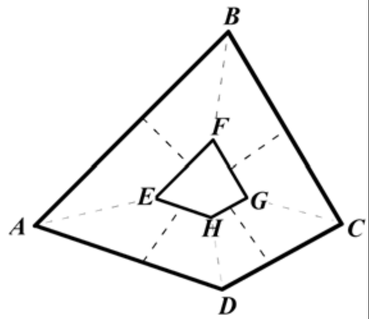
### *7-1: Ratios and Proportions*

<table>
<tr>
<td>A “ratio” of two numbers is a fraction</td>
<td>Ratio of a to b is $\boxed{\frac{a}{b}}$</td>
<td>“to b” means b is in the denominator<br>(that is, divide by b; therefore b ≠ 0).<br>(The archaic notation—sometimes used in the MCAS test—is a:b)</td>
</tr>
<tr>
<td>A “proportion” is an equation that equates two ratios.</td>
<td>$\frac{a}{b} = \frac{c}{d}$</td>
<td>(The archaic notation is a:b :: c:d)</td>
</tr>
<tr>
<td>Put a solution structure on proportion word problems using this 2x2 grid<br><br>This method almost always works. It requires thinking of the problem as describing a pair of objects or events, in each of which two measurements are made.</td>
<td>

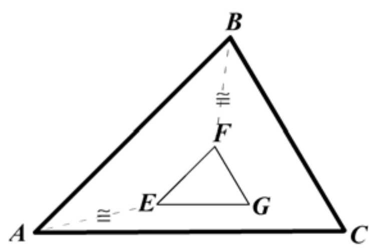
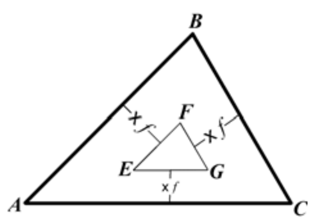
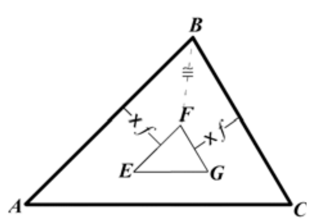
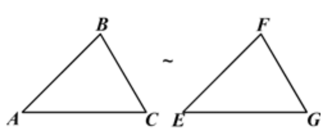
<br>or<br>

</td>
<td>From the 2x2 arrangement to the left, draw two “—”s and an “=” and write<br>$\frac{a}{b} = \frac{c}{d}$<br>This works with:<br>1. shadow problems<br>2. similar triangle problems<br>3. recipe resize problems<br>4. distance-time problems (fixed speed)<br>5. and others<br><br>For triangles, use the second form:<br>


<br>$\frac{a}{b}$ and $\frac{c}{d}$ are both the “scale factor” from U to V.<br>Any side of V times the scale factor gives the corresponding side of U.</td>
</tr>
<tr>
<td>Cross products of a proportion are equal</td>
<td>
</td>
<td>Use this rule to solve for the unknown in a proportion:<br>$\frac{a}{b} = \frac{c}{d}$ becomes $a \cdot d = b \cdot c$</td>
</tr>
</table>

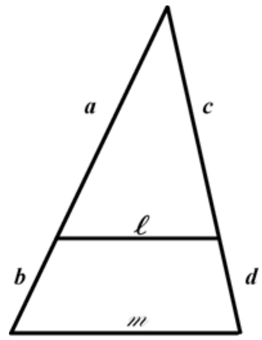
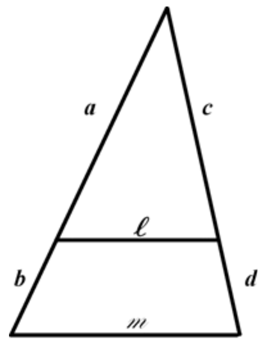
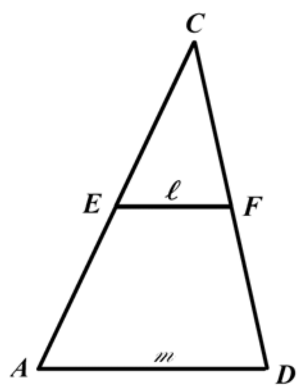
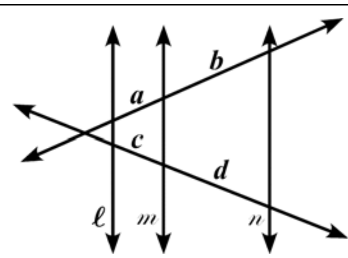
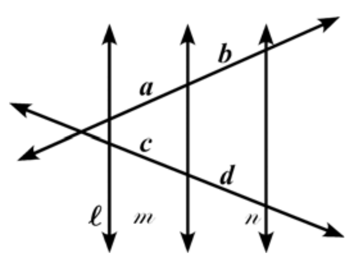
## 7-2: Similar Polygons and Proportions

| | | |
|---|---|---|
| Definition:<br>**Two polygons are similar**<br>**if and only if**<br>**Their corresponding angles are congruent**<br>**and**<br>**Their corresponding sides are proportional.**<br>Similar figures have the same shape but not necessarily the same size.<br>"~" means "is similar to".<br>(Dashed lines indicate correspondence.) |  | "Polygon $ABCD$ **~** Polygon $EFGH$"<br>**The order of the letters indicates the correspondence.**<br>$\angle A \cong \angle E$, $\angle B \cong \angle F$ etc.,<br>The "Scale factor" of<br>$ABCD$ **to** $EFGH$<br>is the *ratio* of the length of *any* side of $ABCD$ **to** the length of the *corresponding* side of $EFGH$.<br>$f = \frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$<br>so $AB = f \cdot EF$ etc.<br>The sides of $ABCD$ are then "**proportional to**" the sides of $EFGH$ |

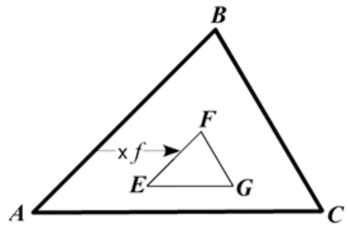
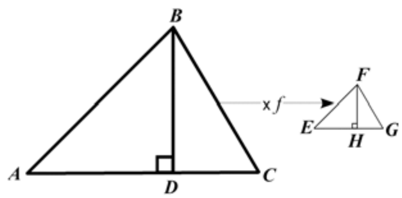
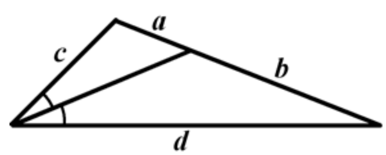
## 7-3: Proving that Triangles are Similar

| | | |
|---|---|---|
| Postulate (AA similarity):<br>**If**<br>**Two angles of one triangle are congruent (respectively) to two angles of a second triangle,**<br>**then**<br>**The triangles are similar** |  | If<br>$\angle A \cong \angle E$ and $\angle B \cong \angle F$<br>then<br>$\Delta ABC \sim \Delta EFG$ |
| Theorem (SSS similarity):<br>**If**<br>**The measures of all corresponding sides of two triangles are proportional**<br>**then**<br>**The triangles are similar** |  | If there is a nonzero $f$ such that<br>$AB = f \cdot EF$ and $BC = f \cdot FG$<br>and $CA = f \cdot GE$<br>then<br>$\Delta ABC \sim \Delta EFG$ |
| Theorem (SAS similarity):<br>**If**<br>**The measures of two sides of a triangle are proportional to the measures of two corresponding sides of a second triangle**<br>**and**<br>**the included angles are congruent**<br>**then**<br>**The triangles are similar** |  | If<br>$AB = f \cdot EF$ and<br>$BC = f \cdot FG$ and<br>$\angle B \cong \angle F$<br>then<br>$\Delta ABC \sim \Delta EFG$ |
| Theorem:<br>**Similarity of triangles is reflexive, symmetric, and transitive** |  | $\Delta ABC \sim \Delta ABC$<br>If $\Delta ABC \sim \Delta EFG$ then $\Delta EFG \sim \Delta ABC$<br>If $\Delta ABC \sim \Delta EFG$ and $\Delta EFG \sim \Delta IJK$<br>Then $\Delta ABC \sim \Delta IJK$ |

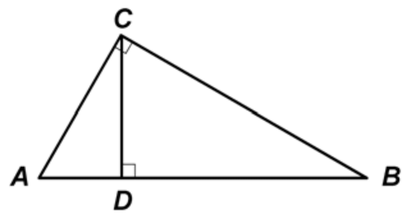
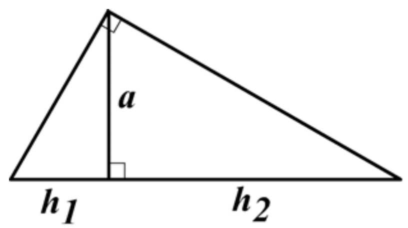
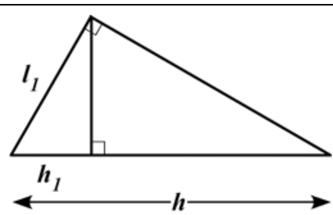
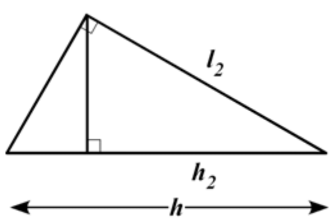
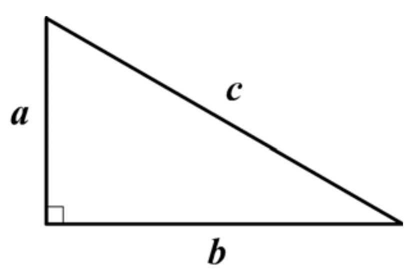
## 7-4: Parallel Lines and Proportional Parts of Triangles

| | | |
|---|---|---|
| Triangle Proportionality Theorem:<br>**If**<br>**a line is parallel to one side of a triangle and intersects the other two sides in two distinct points**<br>**then**<br>**it separates these sides into segments of proportional length.** |  | Given $\ell \parallel m$:<br>Then $\frac{a}{b}=\frac{c}{d}$<br>$\frac{a+b}{b}=\frac{c+d}{d}$ (why?)<br>$\frac{a}{c}=\frac{b}{d}$ (why?) |
| Theorem (converse of above):<br>**If**<br>**a line intersects two sides of a triangle and separates these sides into corresponding segments of proportional lengths**<br>**then**<br>**the line is parallel to the third side.** |  | Given $\frac{a}{b}=\frac{c}{d}$ then $\ell \parallel m$. |
| Theorem:<br>**A segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle and its length is one half the length of the third side.** |  | Given: $E$ is the midpoint of $\overline{AC}$ and F is the midpoint of $\overline{CD}$,<br>Then $\overline{EF} \parallel \overline{AD}$ and $EF=\frac{1}{2}AD$ |
| Corollary:<br>**If**<br>**Three or more parallel lines intersect two transversals**<br>**then**<br>**They cut off the trans-versals proportionally** |  | Given: $\ell \parallel m \parallel n$<br>Then: $\frac{a}{b}=\frac{c}{d}$ |
| Corollary:<br>**If**<br>**Three or more parallel lines cut off congruent segments on one transversal**<br>**then**<br>**They cut off congruent segments on every transversal** |  | Given: $\ell \parallel m \parallel n$ and $a=b$<br>Then: $c=d$ |

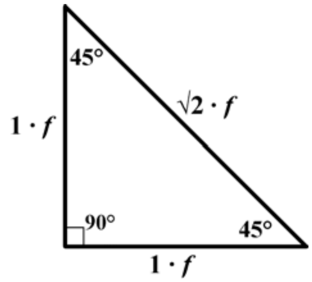
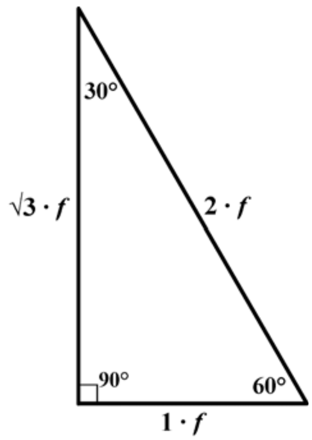
## *7-5: Parts of Similar Triangles*

| | | |
|---|---|---|
| Proportional Perimeters<br>Theorem:<br>**If**<br>**Two triangles are similar**<br>**then**<br>**Their perimeters are proportional to the measures of the corresponding sides** |  | Given: $\Delta ABC \sim \Delta EFG$<br>And $EF = AB \cdot f$ (etc)<br>Then:<br>Perimeter $\Delta EFG$ =<br>Perimeter $\Delta ABC \cdot f$ |
| Three Theorems:<br>**If**<br>**Two triangles are similar**<br>**then**<br>**The measures of their corresponding**<br>**Altitudes**<br>**Medians**<br>**Angle bisectors**<br>**are proportional to the measures of the corresponding sides** |  | Given: $\Delta ABC \sim \Delta EFG$ ,<br>$EF = AB \cdot f$ (etc), and<br>$\overline{BD}$ and $\overline{FH}$ are altitudes (etc)<br>Then<br>$FH = BD \cdot f$ |
| Angle Bisector Theorem:<br>**An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides** |  | $\frac{a}{b} = \frac{c}{d}$ |

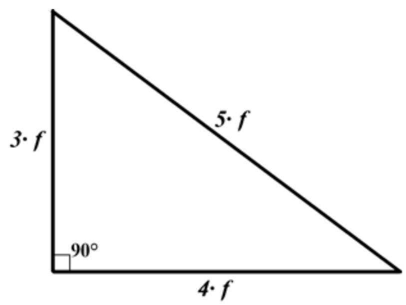
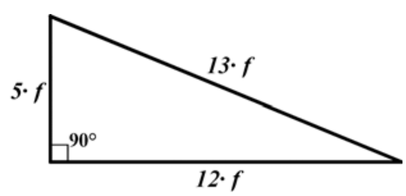
## *Chapter 8: Right Triangles*
## *8-1: Geometric Mean and the Pythagorean Theorem*

| | | |
|---|---|---|
| Theorem 8-1:<br>**If an altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse**<br>**then the two triangles so formed are similar to the given triangle and to each other**. |  | $\Delta ABC \sim \Delta ACD \sim \Delta CBD$ |
| Theorem 8-2:<br>**The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.** |  | $\frac{h_1}{a} = \frac{a}{h_2}$, or<br>$a^2 = h_1 \bullet h_2$, or<br>$a = \sqrt{h_1 \bullet h_2}$ |
| Theorem 8-3:<br>**If an altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse**<br>**then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.** |  | $(l_1)^2 = h \bullet h_1$<br><br>$(l_2)^2 = h \bullet h_2$ |
| **Pythagorean Theorem:**<br>Theorem 8-4:<br>**In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.**<br>**Inverse*:*** Theorem 8-5:<br>***If* the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side**<br>**then the triangle is a right triangle.** |  | $c^2 = a^2 + b^2$ |

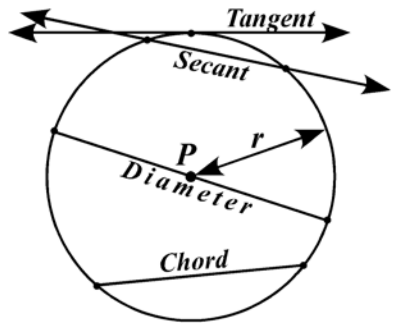
***8-2: Special Right Triangles***

| | | |
|---|---|---|
| Theorem 8-6:<br>**In a 45°-45°-90° triangle, the hypotenuse is √2 times as long as either leg.**<br>This triangle is half a square. |  | ***f*** is the "scale factor". You can make ***f*** be any positive number in order to make the triangle the size you want. |
| Theorem 8-7:<br>**In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is √3 times as long as the shorter leg.**<br>This triangle is half an equilateral triangle. |  | ***f*** is the "scale factor". You can make ***f*** be any positive number in order to make the triangle the size you want. |

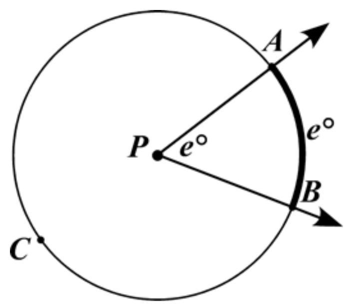
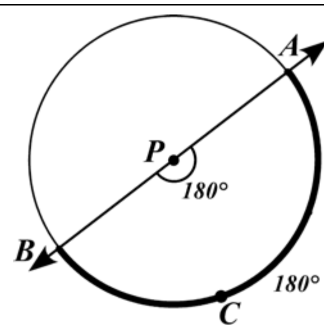
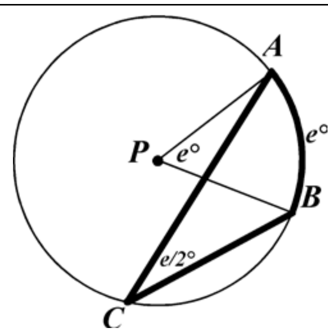
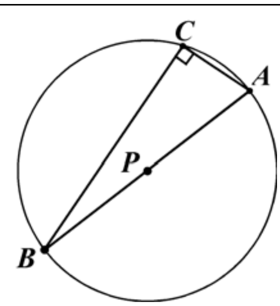
***Addendum: Other important special right triangles***

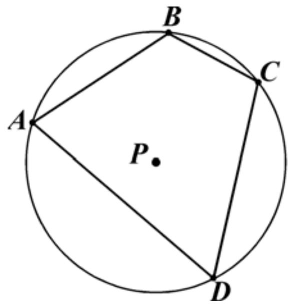
| | | |
|---|---|---|
| **3-4-5 triangle** |  | ***f*** is the "scale factor". You can make ***f*** be any positive number in order to make the triangle the size you want. You will see this triangle in many problems. |
| **5-12-13 triangle** |  | ***f*** is the "scale factor". You can make ***f*** be any positive number in order to make the triangle the size you want. This triangle is just a little less popular than the 3-4-5 triangle in problems. |

## *9-1: Definitions*

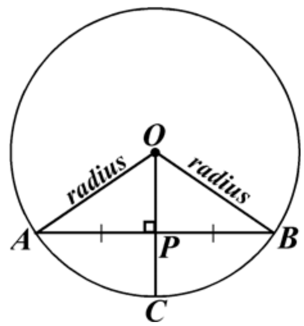
| | | |
|---|---|---|
| **Definitions:**<br>**The set of all points equally distant from a given point is called a circle.** The point *P* is the **center** and the distance *r* is the **radius**. A **chord** is a segment both of whose endpoints are on the circle. A **diameter** is a chord that contains the center. |   | Let *d* be the length of the diameter. Let *C* be the distance around (the *Circumference* of) the circle.<br>$d = 2r$<br>$C = \pi d = 2\pi r$ |

## *9-2: Angles and Arcs, 9-4: Inscribed Angles*

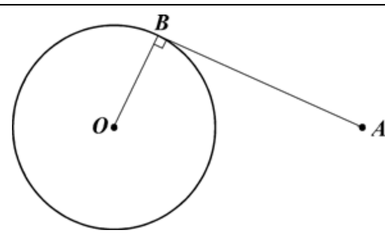
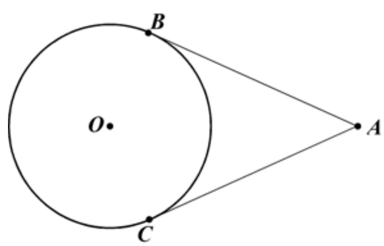
| | | |
|---|---|---|
| **Central angles.**<br>The measure of a **minor arc** is the measure of its central angle. The measure of a **major arc** is 360° minus the measure of its central angle. |   | $\widehat{AB}$ is the *minor arc* intercepted by central angle $\angle APB$<br>$\widehat{ACB}$ is the *major arc* intercepted by central angle $\angle APB$<br>$m\widehat{AB} = e°$<br>$m\widehat{ACB} = 360° - e°$ |
| **Semicircle.**<br>The measure of a **semicircle** is 180°. |   | Segment $\overline{APB}$ is a diameter.<br>$m\angle ABP = m\widehat{ACB} = 180°$<br>$\widehat{ACB}$ is a *semicircle*. |
| **Inscribed angles.**<br>The measure of an **inscribed angle** is one half the measure of its intercepted arc. |   | $\angle APB$ and $\angle ACB$ intercept the same arc $\widehat{AB}$<br>$m\angle ACB = \frac{1}{2} m\angle APB$ |
| **Inscribed right triangle.**<br>Any triangle inscribed in a circle, one of whose sides is a diameter of the circle, is a right triangle. |   | $\Delta ABC$ is inscribed in circle *P*:<br>$m\angle ACB = 90°$<br>if and only if<br>$\overline{AB}$ is a diameter. |

| | | |
|---|---|---|
| Opposite angles of an inscribed quadrilateral are supplementary. |  | Quadrilateral *ABCD* is inscribed in circle *P*.<br>$m\angle A + m\angle C = 180°$<br>$m\angle B + m\angle D = 180°$ |

***9-3: Arcs and Chords***

| | | |
|---|---|---|
| **If a diameter is perpendicular to a chord, then it bisects the chord and its arc.** |  | If $\overline{OC}$ is a radius, $\overline{AB}$ is a chord, and $\overline{OC} \perp \overline{AB}$, then $\Delta OAP$ and $\Delta OBP$ are congruent right triangles. For each triangle, the hypotenuse (e.g., $\overline{OA}$) is a radius, the measure of one leg ($\overline{OP}$) is the distance from the center of the circle to the chord $\overline{AB}$, and the measure of the other leg ($\overline{AP}$) is half the length of the chord. |

***9-5: Tangents***

| | | |
|---|---|---|
| **If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.** |  | $B$ is the point of tangency.<br>Tangent $\overline{AB}$ is perpendicular to radius $\overline{OB}$. |
| **If two segments from the same exterior point are tangent to a circle, then they are congruent.** |  | $\overline{AB}$ and $\overline{AC}$ are tangent to circle $O$.<br>$\overline{AB} \cong \overline{AC}$ |